**European Journal of Science and Theology,** *December 2021, Vol.17, No.6, 129-153*

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# THE KNOWLEDGE OF CLERGY AND LAITY IN THE ORIENTATION OF 9$^{th}$-12$^{th}$ CENTURIES ROMANESQUE CHURCHES

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(Received 16 June 2021, revised 28 July 2021)

## Abstract

The first operation in the laying out of a Christian sacred building is its orientation. The liturgies of the 9$^{th}$-12$^{th}$ centuries determine an E-W alignment. The precision of some churches, with Az very close to 90º and height of the horizon ASS > 0º, indicates the use of geometric methods such as that of Gisemundus (c.800) of the monastery of Ripoll (ACA 106), considered more precise than the Vitruvian methods due to the simplicity of its equinoctial layout. Despite this, there is implicit goniometric knowledge of how to determine the proportion between the gnomon and the shadow related to the 'horologium pedum' (IX-XII) and later azimuthal sundials such as that of Ripoll (ACA 225) (11$^{th}$ century). Gisemundus' method allows this alignment to be plotted with great precision at the equinoxes, using a gnomon of proportion 2/3 and a shadow 1, during the hours 3-9.

*Keywords:* religion, churches, architecture, Gisemundus, Gnomon

## 1. Introduction - the first stone of a sacred building

Nicolas IV (1288-1292) renovated the original façade of Santa Maria Maggiore while he commissioned Filippo Rusuti (cca. 1255-1325) to create mosaics (1288-1292) which he signed *Philippo Rusuti fecit hoc opus*. They depict Pope Liberius (352-366) tracing the ground plan of the basilica on the snow on 5 August 358, with the inscription: *Virgo Maria apparuit PP Liberio dicens: fac mihi ecclesiam in monte Superagio sicut nix indicat* [1]. This image influenced the frontispiece of the work by Paolo de Angelis (1580-1647), *Basilicae S. Mariae Mojoris de Urbe, a Liberio Papa* (1621), depicting the layout of the sacred building [2].

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The importance of the blessing and the subsequent layout of the sacred space was reflected in the *Ceremoniale Episcoporum*, whose prints from the *Pontificale Romanum* (1595) of Clement VIII (1592-1605), referring to *De benedictione & impositione primarii lapidis pro Ecclesi aedificanda (On the blessing and the laying of the first stone for the building)*, depict the figures of the bishop, the deacon and the architect as the main authors of the ceremony [Clementis VII, *Pontificale Romanum Clementis VIII Pont. Max. iussu restitutum atque editum*, Romae, 1595: apud Iacobum Lunam: impensis Leonardi Parasoli & Sociorum]. The rite of this symbolism already appears in *Historia Sancti Florentii Salmurensis*, referred to Geofredo Martel (1040-1060), in which the placement of the cross and the arrangement of the orientation in the church of the castle of Saint-Florent-le-Vieil is described [3].

The beginning of a work - and its symbolic meaning of *Petrus* - begins in the foundations, for whose mission, recognition of the terrain and layout, the figure of the architect is necessary, as recognized by Saint Isidore of Seville (cca. 556-636) in his *Originum sive etymologiarum libri viginti* (cca. 630) (L XIX. viii) [4]. But before the foundations of the sacred building can be laid out, its orientation must be determined by means of a specific axis.

The orientation of Christian sacred buildings during the 9th-12th centuries has been the subject of several studies in Austria [5], Denmark [6], Slovenia [7], England [8, 9] and Italy [10, 11]. In the case of Spain, these have been the Mozarabic churches [12], the pre-Romanesque churches of Asturias [13], those of the Camino de Santiago [14] and the Romanesque of Galicia [15], of the Aran Valley [16] and of the Boi Valley [17].

The duality between the ecclesiastical figure of the abbot or bishop and that of the *magister operis* had engendered in the Augustinian imaginary a specialisation forged in the Romanesque abbeys and cathedrals, whose conceptual division between *theoria* and *practica* in the *De Scientiis* of Domingo de Gundisalvo (fl. 1150), part of the *Catalogue of the sciences of al-Fārābī* (cca. 870-950) [18], is divulged in *Speculum Doctrinale* to the world of cathedrals by Vincent of Beauvais (cca. 1194-1264) [19]. There is little direct evidence during the pre-Romanesque and Romanesque periods (9th-12th century) and everything suggests a unity of knowledge. One type of evidence is the orientation of the sacred buildings proposed by the liturgies of the time, towards the East, which was a real scientific challenge for the time, self-imposed by the clergy. From here, the aim of the research is to provide information on the possible methods for determining the action prior to the construction of a sacred building during the 9th-12th centuries. The exactness of the layout of some churches, with an azimuth Az very close to 90º and horizon height ASS > 0º, makes it necessary to use some kind of geometrical method (Figure 1).

The methodological proposal is based on a synthesis of the different studies carried out in the mountainous region of the Aran Valley (Spain), on the northern slopes of the Pyrenees [20] and in the precision of their layouts [21], especially that of the *Ars gromatica Gisemundi* (cca. 800) (Archivo Corona Aragón - ACA 106) transcribed in the monastery of Ripoll, considered very precise because of the few geometrical operations it requires [22]. The research develops the practical knowledge of geometry and goniometry, such as the relationship between the height of the gnomon (Lg) and the shadows cast (Lu), which are related to the *horologium pedum* known in other Hispanic monasteries (9th-11th centuries). The precision of the orientation is also related to the hour angle (ω) formed by the meridian and postmeridian shadows, which is related to the azimuthal sundials. Such is the case of the one described in the Ripoll (ACA 225) (11th century) developed in this research.

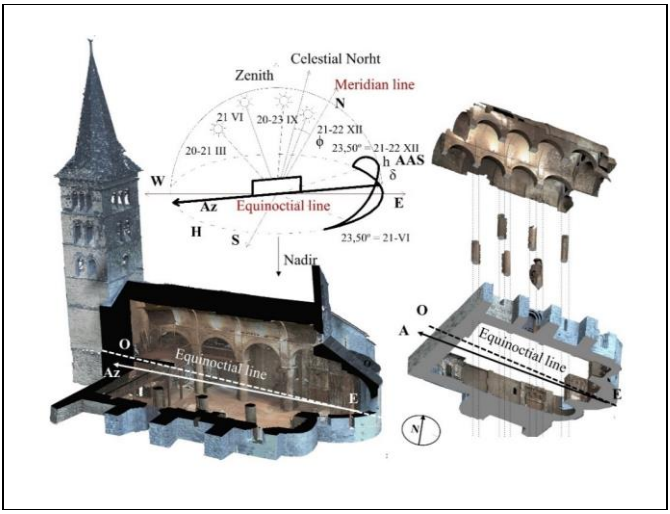

**Figure 1.** Santa María de Arties, Az = 92.12º, ASS = 45.80º (φ = 42.70º).

## 2. Sacred orientation

The search for principles for the canons of medieval creation stem from the cosmology contained in Plato's *Timaeus* (cca. 429-347 BC) and are reflected in what has been defined as *Disegnare il Cosmo, disegnare l'architectura*, through the interaction between the sacred work and the surrounding Universe [23]. Symbolically, the East is the source of light, the direction of Jerusalem; it signifies Paradise, the 'Light of the world' or the 'Sun of righteousness' (Malachi 4.2, Luke 1.78, John 8.12). The East will be the place from which the Redeemer will return to judge mankind. Therefore, the Apostolic Constitutions (Apostolic Const., II, 7) (cca. 380), written by the apostles and handed down by Clement of Rome (+97), point out that the place of meeting and communion must look to the East [24].

In the *Apologeticus* (197) of Tertullian (cca. 160-220) it is said that Christians pray facing east (16.9-11) [25]. The first references to prayer arrangements come from Origenes Adamantius (185-254) in *De Oratione* (cca. 233-234), where he announces that one should pray facing the East [26]. Even before the construction of the early Christian basilicas, the records of the magistrates Hipparchus (+297) and Philotheus (+297), victims of Maximian

(250-310), tell us that in the former’s house there was a space reserved for prayer, on whose wall a cross had been painted to mark the East [27]. Despite this, the first Christian basilicas were built directionally on top of pre-existing buildings. Thus, Saint John Lateran (9-XI-324) is situated on military settlements, Saint Paul Outside the Walls (18-XI-324) at the base of a cemetery near the Tiber (Figure 2), the Constantinian basilica of Saint Peter (cca. 326) takes the orientation of the circus of Nero [28] and Santa Maria la Maggiore (5-VIII-358) the topography and existing buildings on top of the Esquiline Hill, as well as the basilica of the Holy Sepulchre in Jerusalem (335), with direction to Anastasis and all its apses slightly to the west.

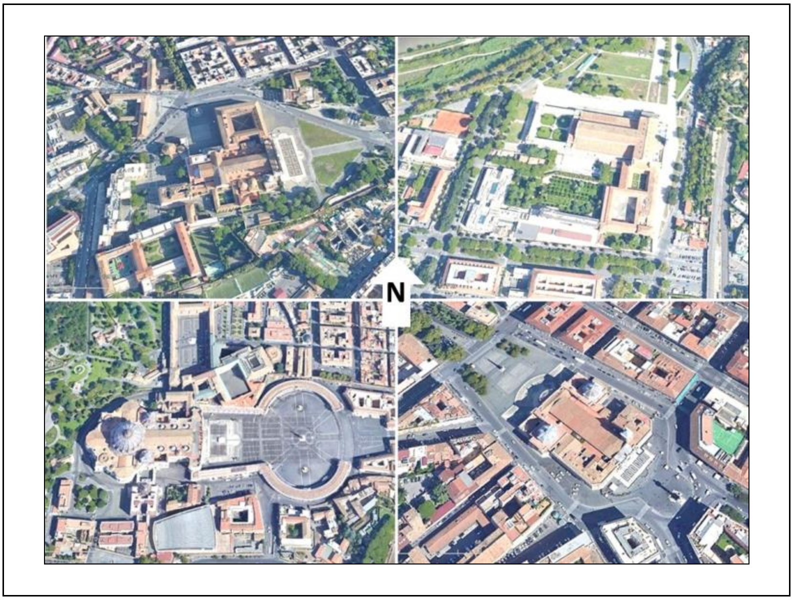

**Figure 2.** Basilicas of Saint John of Lateran, Saint Paul Outside the Walls, Saint Peter the Vatican and Saint Mary Major.

The tradition of the apse to the west continues, according to the documentary sources of Paulin de Nole (353-431), at Saint Felix in Nole, or according to Sidoine Apollinaire (430-486), on a church built in 472 in Lyon [29]. Saint Isidore of Seville (cca. 556-636), in *Originum sive etymologiarum libri viginti* (cca. 630), defines in *De aedificiis sacris*, that the great buildings or *templa* should be aligned with the direction of the equinoctial east [4]. Rabano Mauro (cca. 776-856) will also do so in his *De Universo libri viginti duo* (cca. 844): *orientem expectabant æquinoctialem* (*De. Uni.* L XIV. XXI) [30]. The orientation of the churches evolved: from an orientation with the apses to the west to a reverse orientation with the apses to the east, and from an alignment between the altar and the seat, on the central axis of the church, to the translation of the seat towards the epistle.

### *2.1. The sacred orientations in the 9th-12th centuries*

The study of Christian orientation and archaeoastronomy has revealed that many Mozarabic, pre-Romanesque and Romanesque churches are not oriented on the East-West axis, but that different criteria have been followed for their layout. The canonical criterion, according to the liturgical treatises of the 9th-13th centuries, situates the apses from East to West, inheriting, as M. Delcor indicates, the Roman tradition [31]. That of the Paschal calendar involves churches oriented on the feast of Easter during the year of their construction, as pointed out by G. Romano [32] and S.C. McClusckey [33]. The Liturgical Calendar, according to W. Johnson [34], H. Benson [35] and E. Spinazzè [36], orients the church on the feast day to which it is dedicated. The topographical criterion assumes that the orientation is linked to the geographical environment and was proposed by A.B. Knapp and W. Ashmore [37] and A. Sassin [38]. The constructive criterion, set by the adequacy to the constructive needs, was proposed by J. Pérez Valcárcel and V. Pérez Palmero [39].

The orientation of the churches with the apse towards the East will be imposed thanks to the influence of the main liturgies of the time, based on the *Liber officialis* (820-826) of Amalarius of Metz (cca. 780-851) and which inspired the main Romanesque canons such as that of Rupert of Deutz (c. 1075-1129) and his *Liber de divinis officiis* (1120). These considerations are thus established in the *Gemma animae* (cca. 1120) of Honorius of Autun (1080-cca.1153): *ecclesiae ad orientem vertuntur ubi sol oritur* (Gem. Ani. I, 129, De situ ecclesiae) [40], in the *Mitralis de Officio* (1190) of Sicardo bishop of Cremona (1185-1215): *Ad orientem, id est, ortum solis aequinoctialem (*Mitra. I, 2, *De fundatione ecclesiae)* [41], and in the treatise of Jean Beleth (fl. 1135-1182) *Rationale divinorum officiorum,* (cca. 1150): *Versus sim orientem, hoc est, versus solis ortum aequinoctialem (Ration. cap. II, De loco)* [42].

**Figure 3.** Church of Saint Andreu de Salardú, Equinox (2018), 8:45 h GTM (Google Earth Pro).

However, the determination of the E-W orientation with an azimuth of Az = 90º is only possible if the solar ortho can be visualised on the horizon during the equinoxes, i.e. if ASS = 0º. Otherwise, when the height of the horizon is ASS > 0º, the use of one of the geometrical methods known in the Late Classical world is necessary (Figure 3).

## *2.2. Direct sources of the orientation of late medieval monasteries*

Although there is evidence of clerical and lay knowledge of *De architectura* (cca. 15 BC) by Marcus Vitruvius Pollio (cca. 80-20 BC), its direct influence on medieval architecture is highly questioned [43] and its widespread use would not come about until the discovery of the Poggio codex in the monastery of Saint Gallen around 1415 by Gian Francesco Poggio Bracciolini (1380-1459). Today, 132 codices from up to the 15th century are preserved [44].

The palatine architect Eginardo (770-840), who built a triumphal arch for the monastery of Maestrich, advised understanding and interpreting Vitruvius in his *Epistolae* [45]. The *Constitutio* of York (926) promulgated the study of Euclid and Vitruvius [46]. Bishop Oswald of Morcester (912-992) used geometry for the foundation of Ramsey Abbey (969) [47]. Also, Bishop Bernward of Hildesheim (993-1022) who, quoting Vitruvius, built the abbey church of Saint Michael (1001-1031) [48]. It is also recognised by Riquero of Reims (cca. 940-998) in *Historiae* (CXXXVIII 107): *secundum Vitruvii atque Boetii*, by Hucbaldo of San Amando (840-930) in *De harmonica institutione* (CXXXII 946): *Ut Vitruvius dixit in libro de Architectura*, by Hugo of San Víctor (1096-1141) in *Eruditio Didascalica* (CLXXVI 765): *Palladius quoque De agricultura scripsit; Vitruvius autem De architectura*, by Peter Deacon of Montecasino (1107-1140) in *De viris illustribus Cassinensibus* (CLXXIII 1048): *Vitruvium de Architectura mundi abbreviavit*, or by Vicent of Beauvais (cca. 1194-1264) in *Speculum naturale* (XXVIII 2): *Vitruvius in libro tertio de architectural Corpus hominis ita natura composuit* [49]. Vitruvius was therefore widely recognised. Regarding orientation, Vitruvius describes two methods for determining the meridian from north to south (*Vitr*, LI.VI.6) and the equinoctial from east to west (*Vitr*, LIX.VII.4) [50]. In turn, he claims to know astronomy and the use of the gnomon to determine the equinoxes and solstices (*Vitr*, L.I) [50]. Vitruvius also defined the construction of sundials (*Vitr*, L.IX, 7) [50]. Marcus Caetius Faventinus took up the tradition of the construction of clocks, defining the double-axe or *plecino* clock and the semicircular or quadrant clock (*Fave*. XXIX) [51].

The experience of the Gromatic sources, and their medieval transmission in the monasteries, also allows us to establish other direct sources. Higinio Gromático (fl. 98-102), in *De limitibus constituendi* [52], determines the axes of the course of the Sun, E-W, and the axis of the world, N-S, the origin of which is in the Etruscan tradition; it was based on the point where the Sun rose on the day of the foundation of the city, and which often coincided with the *Die Natalis* of its founder [53]. In *De limitibus*, Frontinus (cca. 30-104) [52, p. 26-33] deals with the orientation of the meridian axes, mentioning the historical references (III.1) and the induced errors (III.5, III.12), but without any description to determine the meridian line [54]. In other cases, the equinoctial orientation is based on rural land division practices applicable to the layout of *castros* or newly created towns using the *varatio* technique [55] of Marcus Iunis Nypsius (f. 200) described in *Fluminis varatio* [52, p. 285] and *Limitis repositio* [52, p. 286-295].

Late-classical scientific knowledge is recorded in the *Scriptorio* of the monastery of Ripoll. Gisemundus' *Ars gromatica* (ACA 106) refers to Varron's geometry, which alludes to the circle of the Earth and its division into months, and reiterates errors made at sunrise and sunset when determining orientations. The 11th century ACA 225 contains the construction of an azimuthal clock [56]. Both codices are related to the apocryphal work of Gerbert of Aurillac (938-1003), the mathematician Pope (999-1003), educated in 986 in Vic by Bishop Ató (957-971), the disseminator of this knowledge in the year 1000.

### 2.3. Methodologies for the layout of the orientations

The layout and precision of the orientation of sacred buildings *ad orientem vertuntur ubi sol oritur* are based on geometrical and observational methods. They trace either the N-S meridian line and then the E-W, or directly the E-W equinoctial line. They are established as coming from those of Vitruvius studied by Moritz Benedikt Cantor (1829-1920) [57]: method $M_1$ (*Vitr*, LI.VI.6) [50, p. 24], which determines the meridian orientation through two shadows with an execution requiring six geometric operations (Figure 4), and method $M_2$ (Vitr, LIX.VII.4) [50, p. 231], which determines an equinoctial orientation through three shadows with an execution requiring fifteen geometric operations.

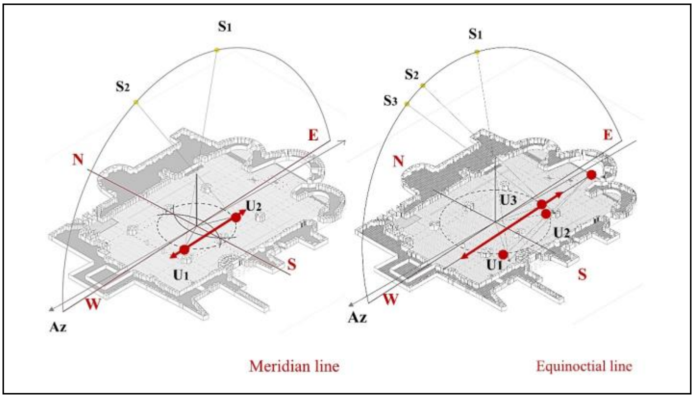

**Figure 4.** Marcus Vitruvius Pollio (c. 15 BC), methods: a) $M_1$, b) $M_2$.

Other techniques come from Higinio Gromaticus' *De limitibus constituendi* [52] and are referred to by Oswald Ashton Wentworth Dilke (1915-1993) [58]. The $M_3$ method [52, p. 108-112] determines the orientation by observing the *hora prima* and the $M_4$ method at the maximum solar altitude during the sixth hour [52, p. 188; 59]. Method $M_5$ [52, p. 188-189] determines a meridian orientation through two shadows. Its execution needs six geometric operations and is similar to Vitruvius' $M_1$, as $M_6$ [52, p. 189-191] will be similar to $M_2$ but determining the equinoctial orientation by means of fifteen operations [60; 60, vol. 3, p. 1376-1377].

Also known are the techniques of the *Ars gromatica siue geometria Gisemundi* (cca. 800), whose $M_7$ procedure is based on the translation of a shadow on a reference circumference in such a way that an equinoctial orientation is determined by two shadow points through four geometric operations (Figure 5). The $M_8$ method obtains an equinoctial orientation using two closely spaced shadows close to noon with a plot of two geometric operations.

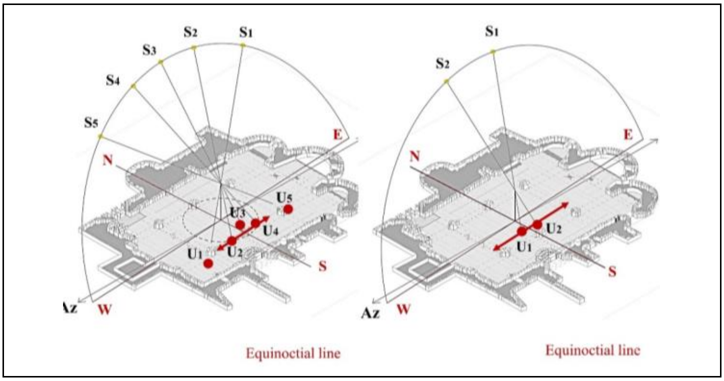

**Figure 5.** *Ars gromatica*, Gisemundus: a) $M_7$, b) $M_8$.

The methods from Gerbert's apocryphal work, *Geometria Incerti Auctoris* (cca. 1000), are $M_9$, similar to Vitruvius' $M_1$ and Higino Gromaticus' $M_5$, and the $M_{10}$ method, similar to $M_2$ and $M_6$ [61]. Finally, one could hypothesise the use of the compass ($M_{11}$), documented in 12th century Europe and known as *Leidarstein*. Alexander Neckam (1157-1217) cites a similar instrument and Guiot de Provins (cca. 1150-1208) speaks of *pierre laide et brunette* [62].

## 3. Precision in the orientation of some Romanesque churches

The mountainous region of the Aran Valley (Spain) is located on the northern slopes of the Pyrenees, bordering the department of Haute-Garonne (France). Its parishes belonged to the diocese of Comenge (France) until 1804, whose chair was occupied by such outstanding ecclesiastical figures as Bertrand de l'Isle (1083-1123) and Bertran de Gòt (1262-1314), appointed Pope Clement V (1305-1314) [63]. At this time, Occitania followed rites of consecration of sacred buildings, as reflected in the case of Saint-Sauveur d'Aux (1103) [64]. Of this period, there are twenty-four churches, fifteen of which (i.e. 62.50%) are oriented from East to West with an interval of 72°-108°, which represents a relative error of 0%-5%. Of these, the respective orientations of Santa Eulària d'Unha, Santa Maria d'Arties, Sant Pèir de Betlan, Sant Andrèu de Casau and Sant Miquèu de Vilamòs have a statistical range of 89.50°-93.32°, with a tolerance of -0.50°, +3.32° with respect to the canonical orientation value of E-W 90°, with a high precision of less than 1% (Table 1).

The eleven methods described above are based on both direct solar observation and an operational system, also based on observation. Analysing the results in Table 2, the root mean square error is $e_c$ = 1.91°, with a statistical range of 89.50°-93.32°. The maximum tolerance ($e_{m,max}$) is 3.32° and therefore the maximum established error of these five churches is only 0.92% (Table 2).

**Table 1.** Azimuth churches equinoctial orientation.

| (Az) azimuth most likely | | | |
|---|---|---|---|
| **Church name** | **Azimuth** | **Azimuth** | **Azimuth** |
| | **(As)** | **(Ag)** | **(Az) = 1/2 (As + Ag)** |
| Santa Eulària d'Unha | 92.58º | 93.15º | 92.87º |
| Santa Maria d'Arties | 91.68º | 92.56º | 92.12º |
| Sant Andrèu de Casau | 90.00º | 90.59º | 90.30º |
| Sant Pèir de Betlan | 92.71º | 92.88º | 92.80º |
| Sant Miquèu de Vilamòs | 90.00º | 87.97º | 88.99º |

**Table 2.** Tolerances of the layout methods of the orientation.

| **Method** | **Author** | **Orientation type** | **Tracing** | **Characteristics** | **Geometric operations** | **Error (max) 0.92%** |
|---|---|---|---|---|---|---|
| M1 | Vitruvius | Meridian | Instrumental | 2 umbra | 6 | 0.15 |
| M2 | Vitruvius | Equinoctial | Instrumental | 3 umbra | 15 | 0.06 |
| M3 | Hyginius Gromaticus | Meridian | Observation | Hora pima | - | - |
| M4 | Hyginius Gromaticus | Equinoctial | Observation | Hora sexta | - | - |
| M5 | Hyginius Gromaticus | Meridian | Instrumental | 2 umbra | 6 | 0.15 |
| M6 | Hyginius Gromaticus | Equinoctial | Instrumental | 3 umbra | 15 | 0.06 |
| M7 | Gisemundus | Equinoctial | Instrumental | 2 umbra | 4 | 0.23 |
| M8 | Gisemundus | Equinoctial | Instrumental | 2 umbra | 2 | 0.46 |
| M9 | Apocrypha of Gerbertus | Meridian | Instrumental | 2 umbra | 6 | 0.15 |
| M10 | Apocrypha of Gerbertus | Equinoctial | Instrumental | 3 umbra | 15 | 0.06 |
| M11 | Compass | Meridian | Observation | Compass | - | - |

### *3.1. Geometric precision in the orientation plot*

The simplest method is Gisemundus' $M_7$, since by directly joining the two points, it determines the equinoctial axis, thus having a smaller margin of error in its plotting ($e_{M7}$ = 0.23) than the $M_1$ and $M_5$ methods. On the other hand, M8, based on two shadows very close to the sixth hour, can give the largest margin of error ($e_{M8}$ = 0.46). The instrumental traces $M_1$, $M_5$, $M_7$ and $M_9$, determined from two shadows - one pre-meridian and one postmeridian - projected on a circumference to ensure that their lengths are equal, have a margin of error of $e_{M1,5}$ = 0.15. The $M_2$, $M_6$ and $M_{10}$ methods use three shadows to determine the orientation by means of rather more complex geometrical procedures, so their complexity would lead them to operate with a much smaller tolerance ($e_{M2,6,10}$ = 0.06).

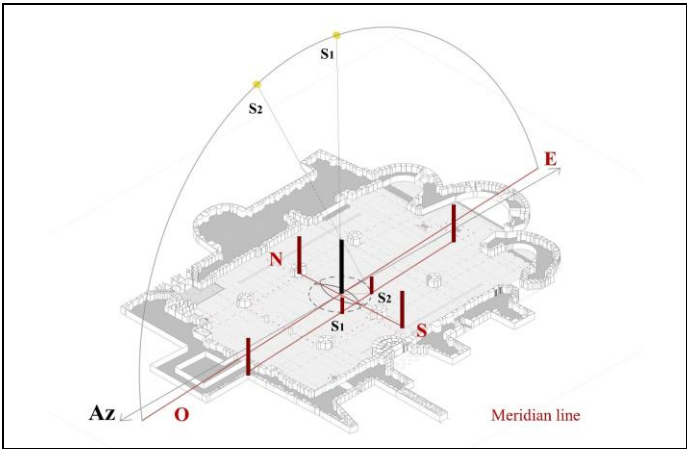

**Figure 6.** *M₁* from Vitruvius, *M₅* from Hyginus Gromaticus and *M₉* of Gerbertus's apocryphal (Santa Maria d'Arties).

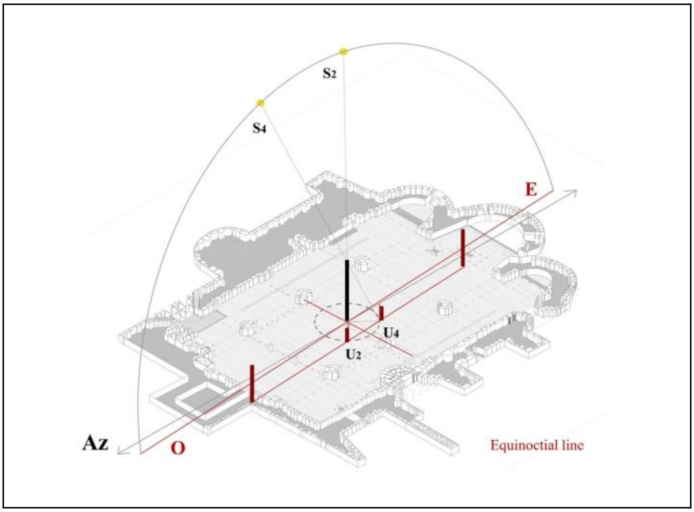

**Figure 7.** Method *M₇* by Gisemundus, setting-out of a church *ad orientem* (Santa Maria d'Arties).

The instruments used for the execution of these methods are referred to in the *Etymologiarum* (XIX.xviii) and transmitted to *De Universo* (XXI.XI). Thus, in $M_1$, $M_5$ and $M_9$, after bisecting the angle, the equinoctial axis would be marked by means of a *pertice aequlite ad perpendiculum*, using a plumb and square, obtaining a tolerance percentage of up to $e_{M1,5}$ = 0.15 (Figure 6). Berand Boysset's *La siensa de atermenar* [65] illustrates how this same axis would have been traced with the rod, making an *extendere linean*, using rods (CBM 327 fol.

251 r), and could have been used in the $M_7$ ($e_{M7} = 0.23$) and $M_8$ ($e_{M8} = 0.46$) methods.

The methods with the greatest tolerance to absorb possible stakeout errors are Gisemundus' $M_7$ and $M_8$, according to which these *ecclesiae ad orientem* should have been staked out either by two shadows, pre- and postmeridian, drawn only with an *extendere lineam*, or aligned, near the sixth hour, by *extendere lineam and pertice aequlite ad perpendiculum*, using a plumb line and a square (Figure 7).

## 4. The 'Ars gromatica siue geometria Gisemundi'

Rudolf Beer (1863-1913) gave the first reference to the *Ars gromatica Gisemundi* in *Die Handschriften des Klosters Santa Maria de Ripoll I*, acknowledging the uniqueness of this treatise on surveying (fol. 76-86) [66] published in the *Boletín de la Real Academia de Buenas Letras de Barcelona* (1910) [67], but it was Carl Olof Thulin (1871-1921) who introduced it in the *Corpus agrimensorum Romanorum* [68] studied and edited by Josep Maria Millàs Vallicrosa (1897-1970) [69].

Two copies of the *Ars gromatica Gisemundi*, whose codicological origins lie in the geometry of Pseudo-Boetius (8th century), have survived: *Codex Parisinus* BN 8812 (cca. 800-833) from southern France (110/097) and *Codex Riuipullensis* 106 (cca. 850-900) from the monastery of Ripoll (118/096) [70]. Its introductory part refers to the *Demonstratio artis geometricae* of Pseudo-Boetius and excerpts from the *De limitibus* of Hyginius Gomatius [71] and the *Corpus Agrimensorum Romanorum*, where the methodology of orientation appears (fol.77r25-77v 10) [72]. The Codex *Ripollensis* of Gisemundus contains traces of Visigothic writing and, from sources close to Higinio Gromático, the work of an early medieval surveyor has been identified who, far from being a mere copyist, knew the basics of this discipline at both a theoretical and practical level [73]. In this context, the $M_7$ method is profoundly practical. The five shadows observed figuratively trace the *analemma* of the solar course. It starts from a geometrical construction of only two basic principles: the tracing of a circumference and the placement of a gnomon squared on the plane where the circumference has been placed, marking directly the equinoctial line, being able to stake out the building directly by means of the operation known as *extendere linean* (Figure 7).

The artificer draws the circumference first thing in the morning, with a diameter smaller than the shadow cast by the gnomon, length $L_g$, at that time. This ($U_1$) is the first geometrical operation to be performed. Then they observe and mark the point where the shadow coincides with the circumference ($U_2$) and wait until the trajectory of the sun approaches the base of the gnomon ($U_3$). Afterwards, marking the point where the end of the shadow coincides again with the circumference ($U_4$), and thus, joining the points ($U_2$-$U_4$), obtains the E-W orientation, without the need to use the last longer shadow ($U_5$).

The $M_7$ construction of the *Ars gromatica Gisemundi* (cca. 800) is eminently practical and is derived from the gromatic experience and the $M_6$ and vitruvian $M_1$ methods, but it operates and aligns the buildings with greater ease and precision. For the final stakeout he will need, after performing *extendere linean*, a new operation by means of *pertice aequlite ad perpendiculum*, with a plumb and square (Figure 6).

### *4.1. Practical Goniometry in the monastery of Ripoll*

Late classical scientific knowledge is contained in the *scriptorio* of the monastery of Ripoll, in Gisemundus' *Ars gromatica*, ACA 106, and in the 11th century ACA 225, both related to the apocryphal work of Gerbert of Aurillac (938-1003), the mathematical pope (999-1003), trained in the year 986 in Vic by Bishop Ató (957-971). Although Gisemundo's $M_8$ method has only two geometric operations, the alignment of the shadows ($U_1$-$U_2$) may have a greater error than the $M_7$ method, since in this case it is possible to draw an alignment from two points further apart ($U_2$-$U_4$), achieving a better alignment and greater precision of orientation. The $M_7$ method is also geometrically very elementary, since it only requires the tracing of a circumference and the observation of the solar course at five points ($U_1$-$U_5$), although it requires certain basic concepts, of goniometric basis, to determine its usefulness and the optimum precision of tracing from the shadows $U_2$-$U_4$. First of all, it will be necessary to know the relationship between the height of the gnomon ($L_g$) and its shadow ($L_u$), which is the radius of the circumference where the shadows cast can intersect. This will depend on the time of year when this method ($M_7$) is used. The shadow of the gnomon is determined by the height of the sun (*Hs*), as a function of latitude (φ) and astronomical declination (δ):

$$\text{Hs} = 90^{\circ} - \varphi + \delta \qquad (1)$$

As a function of the hour angle (h), the solar elevation (θs) at one hour of the day can be determined by the expression:

$$\sin \theta_s = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos h \cdot \cos \delta \qquad (2)$$

A practical consequence of the projection of shadows are the so-called foot clocks by Rutilius Taurus Emilianus Palladius (4th century). By means of tables, the relationship between the shadow of a person referred to the canonical time with respect to the months of the year is expressed. An *Excerpta* of *De temporum ratione* by Bede the Venerable (cca. 672-735) is preserved in Ripoll in ACA 225, ff. 39v-64v, which develops similar questions in the apocryphal work of the *Libellus de Mensura Horologii* in *Concordia XII mensium* [74].

On the other hand, the accuracy depends on the distance between the shadows when they intersect the circumference ($U_2 = U_4$). The further apart these points are, the greater the accuracy of the alignment, and this occurs when the angle between shadows (ω) is closer to 180º, and referred to the hour angle (h) close to the interval +45º - -45º. A practical example of these projections referred to the $M_7$ method is the azimuthal sundial, where the gnomon is not parallel to the axis of the world as in clocks based on the equatorial quadrant, but is placed vertically and perpendicular to the ground; and where the hours are defined by the projection of the shadow according to the height of the sun in the celestial vault and by the angle measured at the horizon, described in the Ripollense codex (ACA 225), ff. 94r-97r.

### *4.2. The gnomon shadow height ratio*

The ratio of the shadow of a body ($L_u$) to the height of a gnomon ($L_g$) is determined by the expression:

$$\text{Tan}\ \theta_s = Lg/Lu \tag{3}$$

The late-classical tradition knew a practical form based on the shadow of a person, tabulated by Rutilius Taurus Emilianus Palladius (4th century) at the end of the books (II-XIII) *De re rustica*, but there is no exact reference to the relationship between the *mensor* and his *umbra in pedes*, neither in the codices nor in the editions of the work. In his edition, Désiré Nisard (1806-1888) attributes to the possibility that this work was written in Gaul [75]. Otto Neugebauer (1899-1990), in his astronomical studies, puts this ratio between 6:1 and 7:1 [60, p. 739]. For his part, Jonathan Roth (1955-), referring to the Imperial Regulations, suggests that the minimum height of recruits is 5+5/12 Roman feet and attributes to them a magnitude of 165 cm [76]. From the point of view of metrology and proportionality, Vitruvius (*Vitr*, LIII.1.7) [50, p. 67] establishes the ratio 6:1, having noted that a man's foot was one sixth of his height and that the elbow consisted of six spans.

**Table 3.** *Horologio* of Rutilius Taurus Emilianus Paladius (4th century).

| **Hour / Month** | **Calendario meses** | | | | | | **Dif col** | **Me/Up 6:1** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Jan | Feb | Mar | Apr | May | Jun | | Jan | Feb | Mar | Apr | May | Jun |
| I-XI | 29 | 27 | 25 | 24 | 23 | 22 | 10 | 4.83 | 4.50 | 4.17 | 4.00 | 3.83 | 3.67 |
| II-X | 19 | 17 | 15 | 14 | 13 | 12 | 3 | 3.17 | 2.83 | 2.50 | 2.33 | 2.17 | 2.00 |
| III-IX | 16 | 14 | 12 | 11 | 10 | 9 | 4 | 2.67 | 2.33 | 2.00 | 1.83 | 1.67 | 1.50 |
| IV-VIII | 12 | 10 | 8 | 7 | 6 | 5 | 2 | 2.00 | 1.67 | 1.33 | 1.17 | 1.00 | 0.83 |
| V-VII | 10 | 8 | 6 | 5 | 4 | 3 | 1 | 1.67 | 1.33 | 1.00 | 0.83 | 0.67 | 0.50 |
| VI | 9 | 7 | 5 | 4 | 3 | 2 | | 1.50 | 1.17 | 0.83 | 0.67 | 0.50 | 0.33 |
| Dif row | 2 | 2 | 1 | 1 | 1 | | | | | | | | |
| | **Me/Up 5,42:1** | | | | | | | **Me/Up 7:1** | | | | | |
| I-XI | 5.35 | 4.98 | 4.61 | 4.43 | 4.24 | 4.06 | | 4.14 | 3.86 | 3.57 | 3.43 | 3.29 | 3.14 |
| II-X | 3.51 | 3.14 | 2.77 | 2.58 | 2.40 | 2.21 | | 2.71 | 2.43 | 2.14 | 2.00 | 1.86 | 1.71 |
| III-IX | 2.95 | 2.58 | 2.21 | 2.03 | 1.85 | 1.66 | | 2.29 | 2.00 | 1.71 | 1.57 | 1.43 | 1.29 |
| IV-VIII | 2.21 | 1.85 | 1.48 | 1.29 | 1.11 | 0.92 | | 1.71 | 1.43 | 1.14 | 1.00 | 0.86 | 0.71 |
| V-VII | 1.85 | 1.48 | 1.11 | 0.92 | 0.74 | 0.55 | | 1.43 | 1.14 | 0.86 | 0.71 | 0.57 | 0.43 |
| VI | 1.66 | 1.29 | 0.92 | 0.74 | 0.55 | 0.37 | | 1.29 | 1.00 | 0.71 | 0.57 | 0.43 | 0.29 |

Methodologically, and given the numerical differences in measurement and proportion, to determine the dimensionality between the height of the observer and the foot, we will take a system of proportionality in the interval 5.42/6/7, so that the proportion between the human body and its shadow is easily observable by the *mensor*. With this, we intend to generalise the use of the same relationship for the gnomon ($L_g$) and its shadow ($L_u$), determining, in a table, the shadow measured in feet, depending on the canonical hour and the month, as well as the different results of proportionality between the height of the *mensor* ($M_e$) and the projected *umbra* ($U_p$), in the case of Palladium (Table 3). Of this type of horary tables, we have direct sources in laudas and ashlars of some Christian churches. A reference chronologically close to Palladius' work is the stone tablet with a foot sundial, although incomplete, in the basilica of Ammaedara (cca. 550), in the Justinian citadel of Haïdra (Tunis), made under the pontificate of bishop Hyacinth of Ammaedara, of which part of the months April/September, May/August and June/July have been preserved. In its reconstruction, the shadows are shortened for the hours of January/December (27-7) and February/November (26-6) [77]. For a latitude $\varphi$ = 35.56º, the interval 0.29-4.98 is close to the human ratio (1/3)-5.

Another such example is the foot sundial at San Pedro de la Nave (1930). The church was moved from the Ricobayo reservoir by Alejandro Ferrant (1897-1976). The panel, engraved on an ashlar measuring 0.72 m. wide by 0.42 m. high, is of uncertain chronology and provenance, and is dated epigraphically to between the 7th and 9th centuries [78]. The inconclusive table shows the months January/December (28-8), and February/November (27/07), with somewhat shorter shadows than those of Palladius. For its present location (φ = 41.58º), the interval 0.29-5.17 is close to the anthropomorphic ratio of (1/3)-5+1/6.

Palladius' work was further refined by other tables according to the latitude of the observer's location and was adapted to the needs and usefulness of the canonical hours of the Benedictine monasteries under the *Regula Sancti Benedicti* (529) of Benedict of Nursia (480-543). The geographical adaptation of these grandfather clocks, as well as the different sources, authors and copies of the codices, have led to a systematic classification: those of series A, or the Aachen model, under the work of the Benedictine Wandalbertus of Prüm (cca. 813-870); those of the B series, or Flavigny model, those classified as C, such as the model of the Abbey of Saint Gallen, of which there will be up to six variants ($C_1$, $C_{1,5}$, $C_2$, $C_C$, $C_L$, $C_S$), under the influence of Bede Venerable (cca. 672-735), and the Mozarabic codices, or series D [79], to which have been added, as E, those of Palladium [80].

In the Iberian Peninsula there are as many as twelve codices described as Hispanic and are found in monastic centres such as San Martin de Albeda (φ = 42.66º) and the *Codex Albeldense Vigilanus* (cca. 976), San Millán de la Cogolla (φ = 42.36º) and the *Codex Aemilianensis* (cca. 992), Santo Domingo de Silos (φ = 41.96º) and the *Liber Ordinum* (cca. 1052). To these are added the antiphonary of the cathedral of León (cca. 950) (φ = 42.60º) and that of the cathedral of Burgo de Osma (11th-12th century) (φ = 41.59º), as well as a copy of the Fuero Juzgo (1058) preserved in the VITR/14/5 of the Biblioteca Nacional de España (Figure 8).

These codices are represented in the shadow tables in which, with a range 0.29-5.17, the summer solstice portion is the same as in Palladio, while the winter portion is less elongated. For the average latitude of the Mozarabic tradition codices (φ = 42º), the human proportion (1/3)-5+1/6 coincides with the *Horologio* of San Pedro de la Nave. Thus, in the Hispanic codices the proportion of the gnomon would have at least a section 1/(5+1/6) (Table 4).

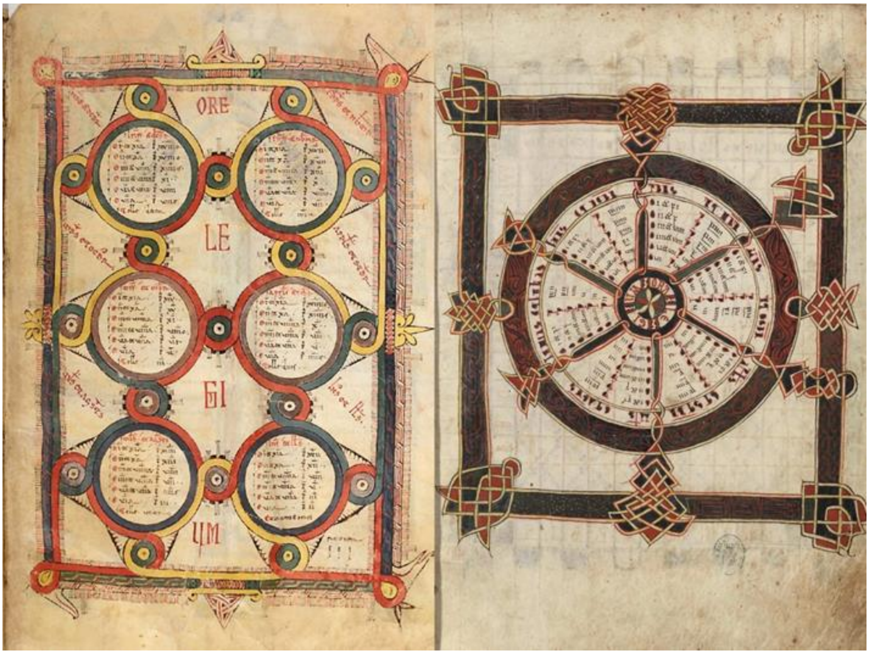

**Figure 8.** Hispanic codices: a) Paris, BnF *Lat*. 2171, f. 23r. (10th century) Vatican; b) Madrid, BNE, *VITR*/14/5, (1058).

**Tabla 4.** Hispanic codex *Horologium pedum*.

| **Horologium Mozarab (VII-XI)** | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Hour / Month** | **Calendar months** | | | | | | **Dif col** | **Me/Up 6:1** | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | | Jan | Feb | Mar | Apr | May | Jun |
| I-XI | 28 | 27 | 25 | 24 | 23 | 22 | 10 | 4.67 | 4.50 | 4.17 | 4.00 | 3.83 | 3.67 |
| II-X | 18 | 17 | 15 | 14 | 13 | 12 | 4 | 3.00 | 2.83 | 2.50 | 2.33 | 2.17 | 2.00 |
| III-IX | 14 | 13 | 11 | 10 | 9 | 8 | 3 | 2.33 | 2.17 | 1.83 | 1.67 | 1.50 | 1.33 |
| IV-VIII | 11 | 10 | 8 | 7 | 6 | 5 | 2 | 1.83 | 1.67 | 1.33 | 1.17 | 1.00 | 0.83 |
| V-VII | 9 | 8 | 6 | 5 | 4 | 3 | 1 | 1.50 | 1.33 | 1.00 | 0.83 | 0.67 | 0.50 |
| VI | 8 | 7 | 5 | 4 | 3 | 2 | | 1.33 | 1.17 | 0.83 | 0.67 | 0.50 | 0.33 |
| Dif row | 1 | 2 | 1 | 1 | 1 | | | | | | | | |
| | **Me/Up 5,42:1** | | | | | | | **Me/Up 7:1** | | | | | |
| I-XI | 5.17 | 4.98 | 4.61 | 4.43 | 4.24 | 4.06 | | 4.00 | 3.86 | 3.57 | 3.43 | 3.29 | 3.14 |
| II-X | 3.32 | 3.14 | 2.77 | 2.58 | 2.40 | 2.21 | | 2.57 | 2.43 | 2.14 | 2.00 | 1.86 | 1.71 |
| III-IX | 2.58 | 2.40 | 2.03 | 1.85 | 1.66 | 1.48 | | 2.00 | 1.86 | 1.57 | 1.43 | 1.29 | 1.14 |
| IV-VIII | 2.03 | 1.85 | 1.48 | 1.29 | 1.11 | 0.92 | | 1.57 | 1.43 | 1.14 | 1.00 | 0.86 | 0.71 |
| V-VII | 1.66 | 1.48 | 1.11 | 0.92 | 0.74 | 0.55 | | 1.29 | 1.14 | 0.86 | 0.71 | 0.57 | 0.43 |
| VI | 1.48 | 1.29 | 0.92 | 0.74 | 0.55 | 0.37 | | 1.14 | 1.00 | 0.71 | 0.57 | 0.43 | 0.29 |

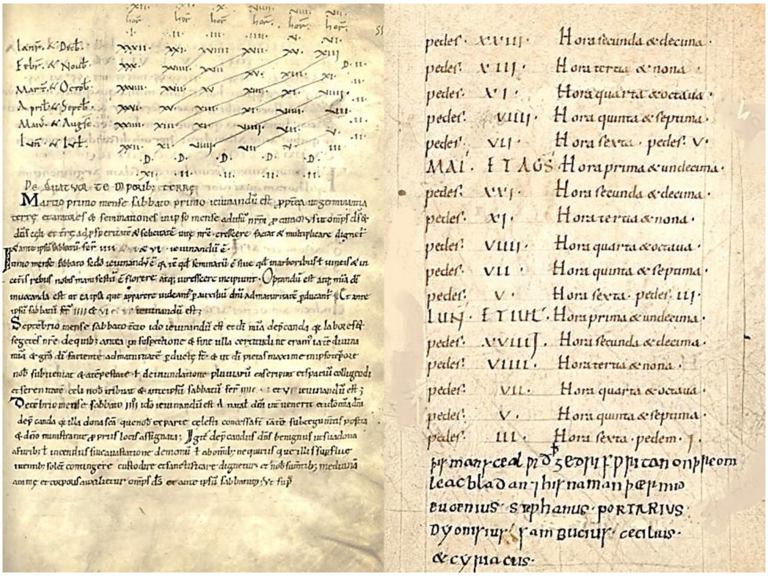

**Figure 9.** Codices A, Madrid, BNE, Mss/3307, códice de Metz, f. 51 r (c. 828-850); Codices C, Vatican, Bibl. Apost. Vat. Reg. lat. 338 f.91 5 (9th century).

The other variants in the surviving *horologium pedum* codices, from series A, B and C, also have representations in the form of tables (Figure 9).

When analysing the set of tables in the codices, for the interval of maximum shadows in the month of January-December at the prime hour (Jan-(I-XI)), and the minimum shadows in the month of June-July at the sixth hour (Jun-(VI)), according to the *concordia mesium*, a result of 0.29-5.90 is obtained (Table 5).

**Table 5.** Medieval Latin Codex *Horologium pedum*.

| **Horologium pedum** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Month/Hour** | **Codex** | | | | | | | | | |
| | **A** | **B** | **C1** | **C1,5** | **C2** | **Cc** | **CL** | **Cs** | **D** | **E** |
| Jan-(I-XI) | 32 | 27 | 29 | 29 | 29 | 29 | 21 | 29 | 28 | 29 |
| Jun-(VI) | 3 | 2 | 1 | 1.5 | 2 | 1.5 | 1 | 2 | 2 | 2 |
| | **Me/Up 5,42:1** | | | | | | | | | |
| Jan-(I-XI) | 5.90 | 4.98 | 5.35 | 5.35 | 5.35 | 5.35 | 3.87 | 5.35 | 5.17 | 5.35 |
| Jun-(VI) | 0.55 | 0.37 | 0.18 | 0.28 | 0.37 | 0.28 | 0.18 | 0.37 | 0.37 | 0.37 |
| | **Me/Up 6:1** | | | | | | | | | |
| Jan-(I-XI) | 5.33 | 4.5 | 4.83 | 4.83 | 4.83 | 4.83 | 3.50 | 4.83 | 4.67 | 4.83 |
| Jun-(VI) | 0.50 | 0.33 | 0.17 | 0.25 | 0.33 | 0.25 | 0.17 | 0.33 | 0.33 | 0.33 |
| | **Me/Up 7:1** | | | | | | | | | |
| Jan-(I-XI) | 4.57 | 3.86 | 4.14 | 4.14 | 4.14 | 4.14 | 3.00 | 4.14 | 4.00 | 4.14 |
| Jun-(VI) | 0.43 | 0.29 | 0.14 | 0.21 | 0.29 | 0.21 | 0.14 | 0.29 | 0.29 | 0.29 |

The minimum leftovers do not affect the ratio in determining $L_u$ to the height of a $L_g$ gnomon, but the longer ones do. The tables of the grandfather clocks give us a ratio for determining the gnomon of [1/6]. With this, we can cover the one used for all the latitudes of the Roman Empire at the time of Vitruvius, $\varphi = 0.23 + 1/4^o - 53 + 1/4^o$.

### *4.3. The horizontal projection of the shadow*

Both the observation of the solar trajectory and the description, on a horizontal plane, of the $M_7$ method of Gisemundus, (cca. 800), have the same basis as the azimuthal sundials. Gerbert's epistle to *Frati Adae* (998) [81], known as *Epistola de horologiis duorum climatum ad fratrem Adam*, is the first direct reference to this type of clock, the typology of which has been preserved in a few Latin codices from the 11th-13th centuries [82] (Figure 10), somewhat later than Gisemundu's work, the earliest being ACA 225, ff. 94r-97r (11th century) [83].

The Ripoll codex describes the construction of two models of the azimuthal clock for a duration of between 9 and 15 hours, drawn on a flat stone [84]. The first, $RIP_1$, is made up of six concentric circles, and described from the inner one with the months of June-July and a duration of 15 h, to the outer circle, January-December, with only 10 h.

The second, $RIP_2$, has seven circles: the first, the month of June, with 15 h; the second, May-July, with 14 h; and so on up to the seventh, December, with 9 h; dividing the figures in both cases by the meridian line. This line allows one to orientate the instrument with the help of the *oroscopo* or polar star. The graphic methodology consists of dividing the day of the month represented in 24 parts, choosing, in $RIP_1$, the circle $C_{1.1}$, marking the hours $H_{11} = 15$, and in $RIP_2$, the circle $C_{2.7}$, marking $H_{2.7} = 9$. The tracing will need certain geometrical and arithmetical skills, since the arcs of circles have to be divided into their respective hours. Thus $Arc_{1.1}$ $H_{11} = 15$ into 15 parts, $Arc_{2.7}$ $H_{2.7} = 9$ into 9 parts, and this, with the help of the astrolabe (Figure 10).

The methodological proposal of the azimuthal clock (11th century) in the Latin West is very similar to the observation of shadows proposed by Gisemundus (9th century). Thus, the layout of these clocks will depend on latitude and, therefore, on daylight hours, with layouts for 9-15 h and 6-18 h. Some codices only mark the time of day. Some codices only mark the first, third and ninth hours, which indicate the change of manual labour, and the sixth hour, which indicates the *Angelus* at midday [85] (Figure 11).

The points of the shadows near the third hour ($U_{III}$), which represents the beginning of the morning work, and the ninth hour ($U_{IX}$), when the afternoon work begins, are the ones that are the furthest apart in their equidistance from the sixth hour ($U_{VI}$), the lunch hour, since they form an hour angle (h) close to $+45^o$, $-45^o$.

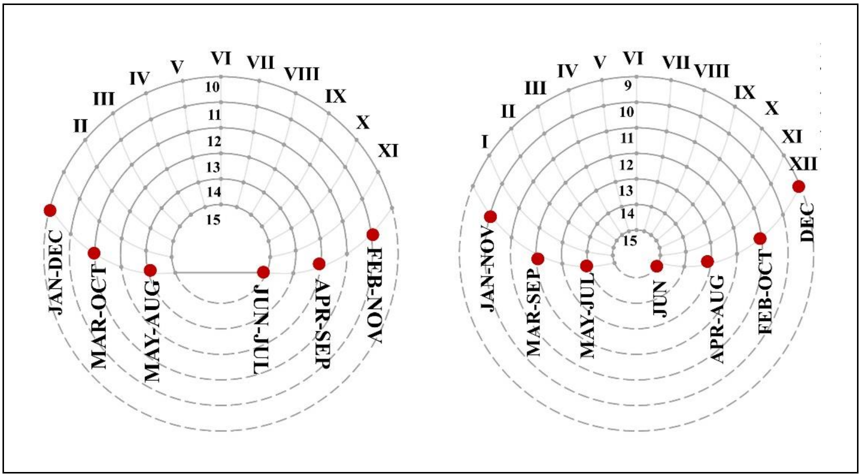

**Figure 10.** Description of the ACA 225 instrument, ff. 94r–97r (11th century).

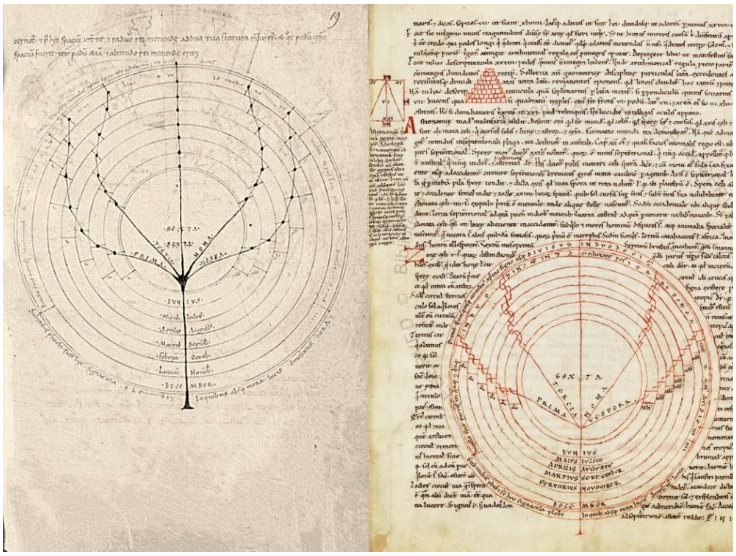

**Figure 11.** Azimuth clocks: a) Paris, BnF *Lat*. 7412, f.19 r. (9th-12th centuries); b) Vatican, Bibl. Apost. Vat., *Lat*. 3101, f. 6v. (13th century).

This knowledge, somewhat later than ACA 106, is implicit in the observation of the $M_7$ method, since the angle ω of the $U_{III}$-$U_{IX}$ shadows is close to 180° and therefore allows for an optimal alignment.

### 4.4. Geometric and goniometric simulation of Gisemundus

Although the text of the ACA 106 does not suggest the construction or instrumentation to determine the orientation according to the $M_7$ method, we can follow an experimental methodology, as Gisemundus would have done, and simulate the process of construction and observation of the Ripoll codex, taking as a reference some later codices for the construction of quadrants in the Muslim West (10th-13th century). We thus take the description of how, with a *balāta* [86] or gnomon, we can know the hours of the day, according to the treatise on the quadrant of the *Kitāb al-hay'a* (11th century) by the Cordovan Qāsim b. Mutarrif al-Qattān [87]. Like ACA 225, the process indicates the arrangement on a soft stone, constructing there a circumference of a span in diameter, with a gnomon in the centre of half a span ($L_u$ 1: $L_g$ ½), placed accurately and without error of inclination, as well as fastening it with lime so that it does not move. The rest of the description is very similar to that of Gisenundus, but tracing the meridian line according to Vitruvius.

**Table 6.** Shadow and its hour angle (ω) of the Ripoll monastery.

| **Ripoll Monastery: (φ) 42º 12' 04'' N, (λ) 2º 11' 27'' E** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Lg/Lu** | **Date** | **Umbra U2** | **Lu = 100** | | | **Δ (Lu2-Lu1)** | | | **ω (L1-L2)º** | **360º- ω** | **(ω L1-L2)/180º** |
| | | **Umbra U4** | **h** | **m** | **sec** | **h** | **m** | **sec** | | | **Ref. = 1** |
| 1 | 21/03/2021 | U2 h(a.m) | 10 | 45 | 0 | | | | | | |
| | | U4 h(p.m) | 13 | 9 | 9 | 2 | 24 | 9 | 72.08 | 287.93 | 0.5996 |
| | 21/06/2021 | U2 h(a.m) | 8 | 33 | 0 | | | | | | |
| | | U4 h(p.m) | 15 | 10 | 20 | 6 | 37 | 20 | 198.67 | 161.33 | -0.1037 |
| | 21/12/2021 | U2 h(a.m) | - | - | - | | | | | | |
| | | U4 h(p.m) | - | - | - | - | - | - | - | - | - |
| 0.75 | 21/03/2021 | U2 h(a.m) | 9 | 32 | 0 | | | | | | |
| | | U4 h(p.m) | 14 | 22 | 5 | 4 | 50 | 5 | 145.04 | 214.96 | 0.1942 |
| | 21/06/2021 | U2 h(a.m) | 7 | 49 | 0 | | | | | | |
| | | U4 h(p.m) | 15 | 54 | 26 | 8 | 5 | 26 | 242.72 | 117.28 | -0.3484 |
| | 21/12/2021 | U2 h(a.m) | - | - | - | | | | | | |
| | | U4 h(p.m) | - | - | - | - | - | - | - | - | - |
| 0.67 | 21/03/2021 | U2 h(a.m) | 9 | 9 | 50 | | | | | | |
| | | U4 h(p.m) | 14 | 44 | 24 | 5 | 34 | 34 | 167.28 | 192.72 | 0.0706 |
| | 21/06/2021 | U2 h(a.m) | 7 | 31 | 51 | | | | | | |
| | | U4 h(p.m) | 16 | 11 | 26 | 8 | 39 | 35 | 259.79 | 100.21 | -0.4433 |
| | 22/12/2021 | U2 h(a.m) | - | - | - | | | | | | |
| | | U4 h(p.m) | - | - | - | - | - | - | - | - | - |
| 0.5 | 21/03/2021 | U2 h(a.m) | 8 | 24 | 34 | | | | | | |
| | | U4 h(p.m) | 15 | 29 | 43 | 7 | 5 | 9 | 212.50 | 147.50 | -0.1806 |
| | 21/06/2021 | U2 h(a.m) | 6 | 53 | 8 | | | | | | |
| | | U4 h(p.m) | 16 | 50 | 15 | 9 | 57 | 7 | 298.56 | 61.44 | -0.6587 |
| | 22/12/2021 | U2 h(a.m) | - | - | - | | | | | | |
| | | U4 h(p.m) | - | - | - | - | - | - | - | - | - |
| 0.25 | 21/03/2021 | U2 h(a.m) | 7 | 12 | 42 | | | | | 76.04 | |
| | | U4 h(p.m) | 16 | 41 | 37 | 9 | 28 | 55 | 284.46 | 75.54 | -0.5803 |
| | 21/06/2021 | U2 h(a.m) | 5 | 43 | 23 | | | | | | |
| | | U4 h(p.m) | 18 | 0 | 0 | 12 | 14 | 37 | 367.31 | -7.31 | -1.0406 |
| | 22/12/2021 | U2 h(a.m) | 9 | 2 | 8 | | | | | | |
| | | U4 h(p.m) | 14 | 33 | 30 | 5 | 31 | 22 | 165.68 | 194.32 | 0.0795 |

The instrumentation necessary for Gisemundus' method appears in the Christian encyclopaedists and in *De instrumentis aedificiorum* and *De Lignariis of the Etymologiarum* (XIX.18-19), according to which, the circumference is traced with the compass, *circinus*, (*Etym*, XIX.19.10), or with the cord, *linea*

(*Etym*. XIX.18.3). The square, *norma* (*Etym*.XIX.18.1) or the plumb line, *perpendiculum* (*Etym*. XIX.18.2) is used to fix the gnomon and, finally, the cord or the ruler, *regula* (*Etym*. XIX.18.2), for the alignment of the coincident points.
For the simulation, a circumference of radius R = 1, $L_u$ = 1, was constructed with a gnomon of different proportions according to the proportional system of measurement of the time ($L_{g.}$): the square ratio (1:1), the *sesquitertia* (1:3/4), the *sesquitertia* (1:2/3), the half (1:1/2) and the quarter (1:4), at the coordinates of the monastery of Ripoll (42º 12' 04" N - 2º 11' 27" E). The shadows ($U_i$) were determined for a mean hour ($h_i$), with a GTM = 0 to facilitate their reproduction; although the graphic results will not be symmetrical over the solar angle h = 0º. Of the five shadows $U_i$ described by the method, two intersect the circumference, the first one ($U_2$) in the pre-meridian hour ($h_2$), and the other one ($U_4$), in the postmeridian hour ($h_4$), forming an hour angle (ω) (Table 6).

The graphical results of the application of the Gisemundo method are determined for the mean time, represented by (12) in the figure. Placing the local time on the perpendicular of the shadows U2-U4 and the local solar angle h = 0º (VI), it can be determined that the high gnomons, of ratio 1:1, have higher geometric accuracy at the summer solstice, since at this time of the year the shadows $U_2$-$U_4$ reach an angle ω = 180º. This ratio does not allow the method to be used during the winter solstice (Figure 12).

Gnomons of the shortest measurement interval (3:4, 1:2) facilitate their use during the equinoxes. However, they are also not useful during the winter solstice and the days around it, when at least one gnomon of experienced proportions of 1:4, 0.25 in Table 5, would be needed, which would allow two points close to the meridian line to be plotted with a ω = 165.68º; thus, short gnomons are more effective during the winter solstices. The optimal E-W plotting for the Ripoll monastery occurs when the alignment of the shadows $U_2$-$U_4$, with an angle ω = 167, 28º, very close to the hour angle (h) +45º, -45º, which represent the canonical hours (III-IX), occurs with the ratio of 2:3 at the equinoxes.

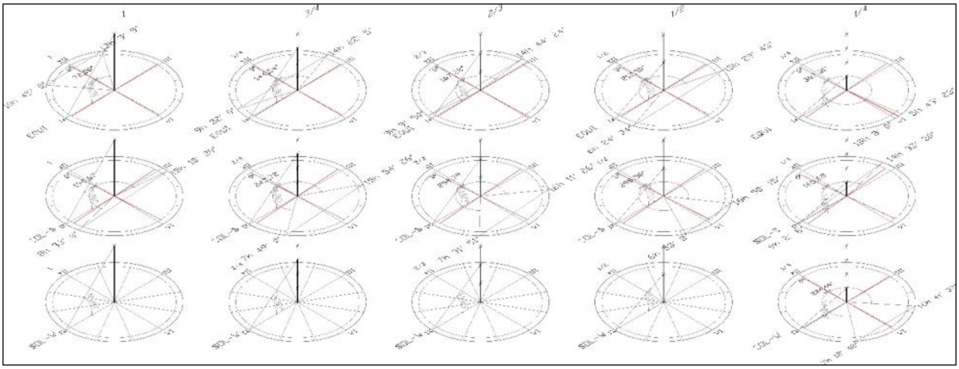

**Figure 12.** Application of the Gisemundus' method ($M_7$), in the solstice equinoxes for different lengths of gnomos.

## 5. Conclusions

In the Spanish Pyrenees monastery on the edge of the Andalusian culture, the monastery of Ripoll transcribed some scientific works of the late-classical culture, collecting the late medieval sources and preserving part of the geometrical knowledge. This was decisive in the formation of Gerbert of Aurillac, who was to become pope in the year 1000. According to the liturgies from the *Liber officialis* (820-826) of Amalarius of Metz and, especially, the *Liber de divinis officiis* (1120) of Rupert of Deutz, sacred constructions must have an equinoctial orientation, according to sources that would have to be sought in the scarce redoubts of Roman and Byzantine knowledge. Gisemundus' methods were born of the needs of surveying, not of liturgy, but in view of the scarcity of resources, they were used in the latter as well as, inductively, in the determination, with a certain precision, of the time patterns of monastic life.

The methods of the *Ars gromatica Gisemundi* (cca. 800) are the simplest from the geometrical point of view as they trace directly the E-W direction. Moreover, they were known to the copyists of southern France (cca. 800-833) and Ripoll (cca. 850-900), being a practical re-adaptation of Vitruvius' meridian method. The $M_7$ construction of the *Ars gromatica Gisemundi* (cca. 800) is eminently practical. Its theory, not described by the copyists, is based on the principles of Marcus Vitruvius Pollio, who claims to know astronomy and the use of the gnomon to determine the equinoxes and solstices (*Vitr*, L.I) [50].

The *Ars gromatica siue geometria Gisemundi* takes up a geometric simplification of Vitruvius' method and directly draws the equinoctial line with only four graphic operations: a circumference, two points and an alignment, directly constructing the equinoctial orientation. To do so, he has to solve, implicitly in the process, some goniometric questions such as the proportion between the radius of circumference Lu and the shadow with respect to the height of a gnomon $L_g$. This question had already been addressed by Roman surveying through Palladius and had been noted in the *scriptorios* of the monasteries, from the 8th century onwards, in the tables of the *horologium pedum*. High gnomons (1:1) work best for the summer solstice, low gnomons (1:1/4) for the winter solstice and medium gnomons (2:3) for the equinoxes. The accuracy will depend on the projection of shadows, an issue later developed in azimuthal sundials. Therefore, when the shadows ($U_2 = U_4$) are close to the hours III-IX, the alignment is best drawn. This time of day coincides with the time when, in monastic life, the '*ora et labora*' work is changed.

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